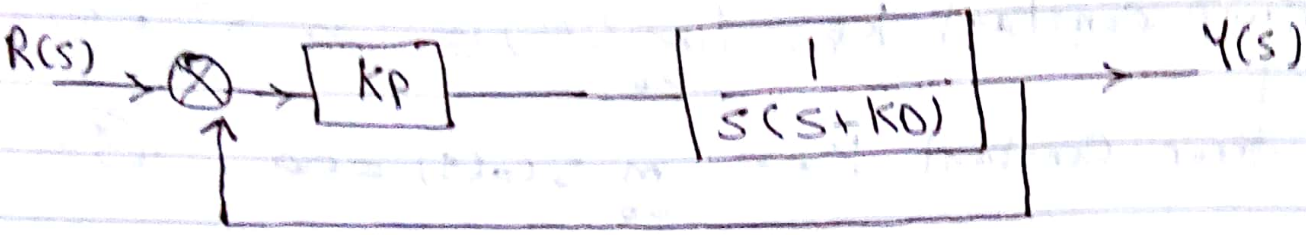
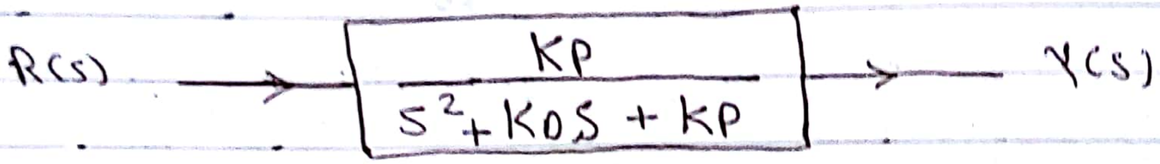
Question 1).

a) * Simplify to get the C.L transfere Function (5 Marks)





(2)

- The General form for T.F for 2nd order system

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\therefore$ $K_P = \omega_n^2$ and $K_D = 2\xi\omega_n$

$\Rightarrow$ setling time $(t_s) = 0.5$ $\Rightarrow$ $t_s = \dfrac{4}{\xi\omega_n} = 0.5$

$\therefore$ $\xi\omega_n = 8$ $\therefore$ $\boxed{K_D = 2\xi\omega_n = 16}$ (1.5)

$\rightarrow$ Maximum overshoot $M_P\% = 20\% = \exp\left(-\left(\dfrac{\pi\xi}{\sqrt{1-\xi^2}}\right)\right) = 0.2$

$\therefore$ $\dfrac{\cancel{-}\pi\xi}{\sqrt{1-\xi^2}} = \ln 0.2 = \cancel{-}1.61 \Rightarrow \xi = 0.456$

$\omega_n = \dfrac{8}{\xi} = \dfrac{8}{0.456} = 17.54 \Rightarrow \boxed{K_P = \omega_n^2 = 307.8}$ (1.5)

b) (5 marks)

$$G(s) = \frac{K(1+2s)(1+4s)}{s^2(s^2+s+1)}$$

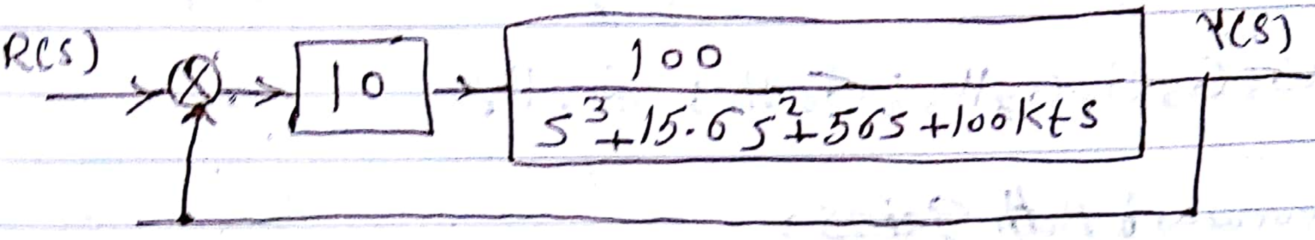
step error constant $K_p = \lim_{s \to 0} G(s) = \infty$ (1.5)

ramp error constant $K_v = \lim_{s \to 0} s\,G(s) = \infty$ (1.5)

Parabolic error constant $K_a = \lim_{s \to 0} s^2 G(s) = K$ (2)

Question 2 }

a) • Simplify The block diagram to the simplest form



$$\frac{Y(s)}{R(s)} = \frac{1000}{s^3+15.6s^2+(100K_t+56)s+1000}$$

• The characteristics equation is

$s^3+15.6s^2+(100K_t+56)s+1000$ ②

• Applying Routh's stability cretirion:

s^3	1	$100K_t+56$
s^2	15.6	1000
s^1	$100K_t \cancel{+49.56}$ -8.1	0
s^0	100	

$\boxed{100K_t - 8.1}$

• to acheive stability $(100K_t + 8.10)$ must be positive

$100K_t + 48.16 > 0$

$K_t > \cancel{-0.4956}\ 0.08$ ∴ $K_{tc} = \cancel{-0.4956}\ 0.081$ ②

• to determine The frequency

$15.6s^2 + 1000 \Rightarrow s^2 = -64.1$

$s = \pm j8 = \pm j\omega$

$\omega = 8$ rad/s

$f = 0.785$ Hz ①

b) – Closed-loop gains: (5 marks)

$G_2G_4G_6G_7H_3$; $G_2G_5G_6G_7H_3$; $G_3G_4G_6G_7H_3$

$G_3G_5G_6G_7H_3$; G_6H_1; G_7H_2

– Forward path gains:

$T_1 = G_1G_2G_4G_6G_7$; $T_2 = G_1G_2G_5G_6G_7$

$T_3 = G_1G_3G_4G_6G_7$, $T_4 = G_1G_3G_5G_6G_7$

– Non toching loops 2 at a time: $G_6H_1G_7H_2$ (2)

$$\Delta = 1 - [H_3G_6G_7(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5) + G_6H_1 + G_7H_2]$$
$$+ [G_6H_1G_7H_2]$$

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$$

$$T(s) = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta} \quad (1)$$

$$= \frac{G_1G_2G_4G_6G_7 + G_1G_2G_5G_6G_7 + G_1G_3G_4G_6G_7 + G_1G_3G_5G_6G_7}{1 - H_3G_6G_7(G_2G_4 + G_2G_3 + G_3G_4 + G_3G_5) - G_6H_1 - G_7H_2 + G_6H_1G_7H_2}$$

(2)

Question 3) [10 Marks]

- A root locus exists on the real axis between the origin and $-\infty$. The angles of asymptotes of the root locus branches are obtained as follow

$$\theta_{asy} = \frac{\pm 180(2K+1)}{3} = \boxed{60^\circ, -60^\circ, -180^\circ}$$ (2)

- The intersection of asymptotes on real axis

$$\sigma_c = \frac{0-2-2}{3} = \boxed{-1.333}$$ (2)

- The breakaway and break-in points are found from $\frac{dK}{ds} = 0$

$$s^3 + 4s^2 + 5s + K = 0$$

$$K = -(s^3 + 4s^2 + 5s)$$

$$\frac{dK}{ds} = -(3s^2 + 8s + 5) = 0$$

$$\boxed{\begin{matrix} s = -1; & \text{and } s = -1.6667 \\ K = 2; & K = 1.852 \end{matrix}}$$ [illegible] (2)

- The angle of Departure

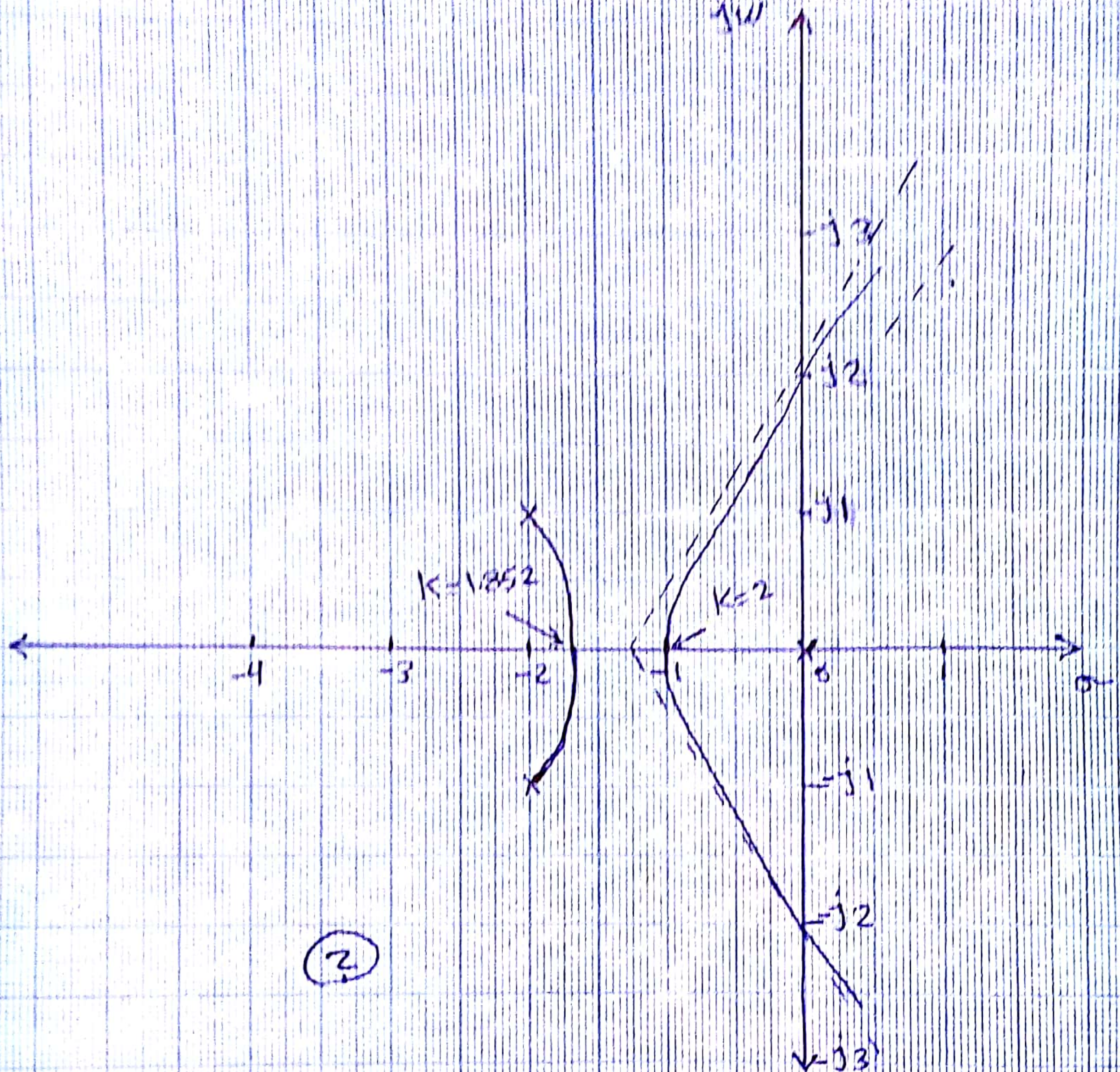
$$\theta_D = 180^\circ - 153.43 - 90^\circ$$

$$\boxed{\theta = -63.43^\circ}$$ (1)

- $(jw)^3 + 4(jw)^2 + 5(jw) + K = 0$

$$(K - 4w^2) + jw(5 - w^2) = 0$$

$$\boxed{\begin{matrix} w = \pm\sqrt{5} & K = 20 \\ w = 0 & K = 0 \end{matrix}}$$ (1)



jω
j3
j2
j1
-j1
-j2
-j3
K=1.852
K=2
-4
-3
-2
-1
0
σ
2